

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc., B.COM DEGREE EXAMINATION – MATHS, PHYSICS & COMMERCE

THIRD SEMESTER – NOVEMBER 2013

ST 3205/3202 - ADVANCED STATISTICAL METHODS

Date : 13/11/2013
Time : 9:00 - 12:00

Dept. No.

Max. : 100 Marks

PART – A

Answer all the questions

10 x 2 = 20 Marks

1. Write down the class frequencies of all orders in case of 3 attributes A,B and C.
2. Provide the conditions for consistency of data involving three attributes.
3. Check whether A and B are independent for the following data:
 $(AB) = 256$, $(\alpha B) = 768$, $(A\beta) = 48$ and $(\alpha\beta) = 144$
4. Define Yule's coefficient of association and coefficient of colligation.
5. If $(AB) = 2340$, $(A\beta) = 230$, $(\alpha B) = 260$ and $(\alpha\beta) = 2340$ find the other class frequencies.
6. Write the sample space for the experiment of tossing three fair coins.
7. Define normal distribution.
8. If X has the probability mass function
 $f(x) = q^x p$, $x = 0,1,2 \dots$, $0 < p \leq 1$; $f(x) = 0$, otherwise
Compute $E(X)$.
9. Write any two uses of chi-square statistic.
10. Write a note on mean and range control charts.

PART – B

Answer any five questions

5 x 8 = 40 Marks

11. Show that for n attributes A_1, A_2, \dots, A_n
 $(A_1 A_2 \dots A_n) \geq (A_1) + (A_2) + \dots + (A_n) - (n-1) N$, where N is the total number of observations.
12. If $\delta = (AB) - (AB)_0$ then with usual notations prove that
 $[(A) - (\alpha)] [(B) - (\beta)] + 2N\delta = (AB)^2 + (\alpha\beta)^2 - (A\beta)^2 - (\alpha B)^2$.
13. State and prove Boole's inequality.
14. (a) If A_1, A_2, \dots, A_n are independent events with $P(A_i) = 1 - (1/\alpha^i)$, $i=1,2, \dots, n$, find the value of $P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)$.
(b) Suppose the events A_1, A_2, \dots, A_n are independent and that $P(A_i) = 1/(i+1)$ for $1 \leq i \leq n$ find the Probability that none of the n events occurs. (4+4)

...2

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15. A random variable X has the following probability distribution :

| | | | | | | | | | |
|--------|---|----|----|----|----|-----|-----|-----|-----|
| X=x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| P(x) : | k | 3k | 5k | 7k | 9k | 11k | 13k | 15k | 17k |

- (i) Determine the value of k.
(ii) Find $P(X < 3)$, $P(X \geq 3)$ and $P(0 < X < 5)$.

16. If X has the probability mass function

$$P(x) = e^{-\lambda} \lambda^x / x! , x = 0, 1, 2 \dots , \lambda > 0 , \text{ find mean and variance of X.}$$

17. Ten individuals were chosen at random from a normal population and their heights were found to be 63, 63, 66, 67, 68, 69, 70, 71, 71 inches. Test if the sample belongs to the population whose mean height is 66". Use 5% level of significance.

18. The following data give the number of defectives in 10 independent samples of varying sizes from a production process:

| | | | | | | | | | | | |
|--------------------|---|------|------|------|------|------|------|------|------|------|------|
| Sample No. | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Sample size | : | 2000 | 1500 | 1400 | 1350 | 1250 | 1760 | 1875 | 1955 | 3125 | 1575 |
| No. of defectives: | | 425 | 430 | 216 | 341 | 225 | 322 | 280 | 306 | 337 | 305 |

Draw the control chart for fraction defective and comment on it.

PART – C

Answer any two questions

2 x 20 = 40 marks

19. (a) Find the remaining class frequencies given the following data:

$$N = 23713 , (A) = 1618 , (B) = 2015 , (C) = 770 , (AB) = 587 , (AC) = 428 , (BC) = 335 \text{ and } (ABC) = 156.$$

(b) If Q and Y denote the Yule's coefficient of association and coefficient of colligation respectively,

$$\text{Show that } Q = 2Y / (1 + Y^2).$$

(15 + 5)

20 (a) State and prove Bayes' theorem.

(b) Three urns I, II and III contain marbles as follows:

4 white , 5 black and 3 red marbles

2 white , 1 black and 1 red marbles

1 white , 2 black and 3 red marbles.

One urn was chosen at random and two marbles were drawn from it. They were found to be white and red . What is the probability that they have come from urn I, urn II or urn III ?

(c) If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{8}$, find (i) $P(A | B)$ (ii) $P(B | A)$

(iii) $P(A^c | B)$ (iv) $P(A | B^c)$ (v) $P(A^c | B^c)$ (vi) $P(B^c | A)$ (vii) $P(B | A^c)$

(4+9+7)

...3

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21 (a) The mean yield for one acre plot is 662 kgs with a standard deviation of 32 kgs.

Assuming normal distribution how many one-acre plots in a batch of 1200 plots would you expect to have yield (i) over 700 kgs (ii) below 650 kgs (iii) what is the lowest yield of the best 100 plots ?

(b) Fit a Poisson distribution to the following data which gives the number of doddens in a sample of Clover seeds:

| | | | | | | | | | | |
|--------------------|---|----|-----|-----|----|----|----|---|---|---|
| No. of doddens | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Observed frequency | : | 56 | 156 | 132 | 92 | 37 | 22 | 4 | 0 | 1 |

Also test the goodness of fit at 5% level of significance.

(7 + 13)

22 (a) In a large city A , 20 percent of a random sample of 900 school children had defective

eye-sight. In another large city B, 15 percent of a random sample of 1600 children had the same defect. Is this difference between the two proportions significant? Use 1% level of significance.

- (b) Four experimenters determine the moisture content of samples of powder, each man taking a sample from each of six consignments. The assessments are :

| Observer | Consignment | | | | | |
|----------|-------------|----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 9 | 10 | 9 | 10 | 11 | 11 |
| 2 | 12 | 11 | 9 | 11 | 10 | 10 |
| 3 | 11 | 10 | 10 | 12 | 11 | 10 |
| 4 | 12 | 13 | 11 | 14 | 12 | 10 |

Carry out the ANOVA and discuss whether there is any significant difference between consignments and between observers. Use 5% significance level.

(5 + 15)